

EULER-IV

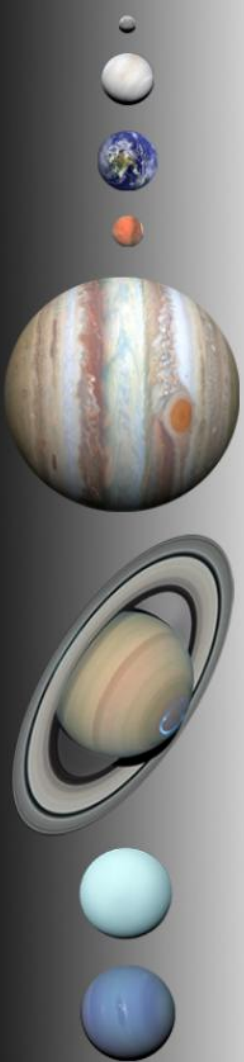
**DYNAMICALLY WARM CO-ORBITALS OF THE EARTH:
A SEMI-ANALYTICAL THEORY**

V.V.Sidorenko (KIAM), N.Adigozalov (MIPT)

**Saint-Petersburg
August 21, 2026**

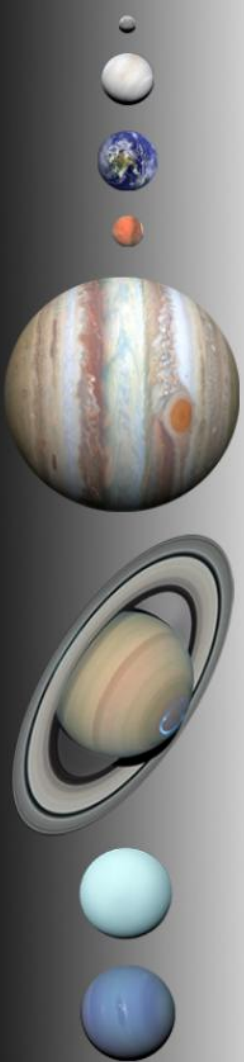
OUTLINE

- Coorbital motions: introduction
- Adiabatic approximation in studies of resonant motions: main ideas
- Application of AA for studies of coorbital motions
- Conclusions

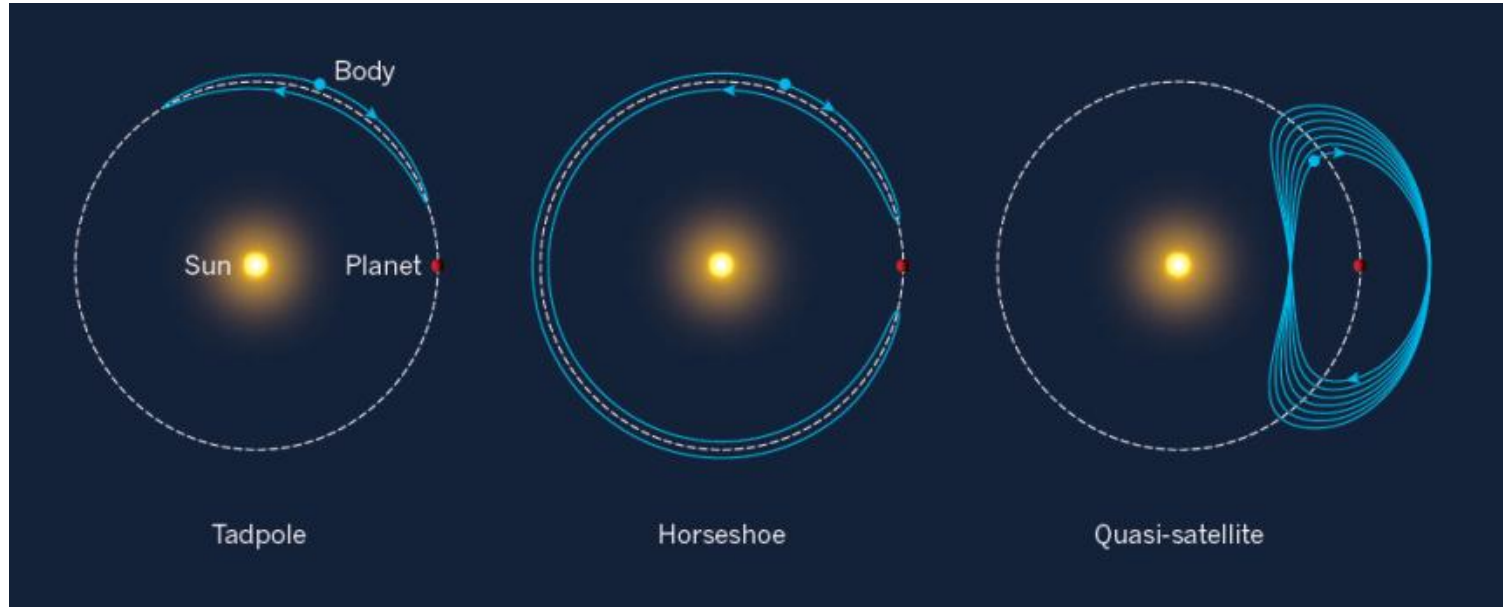


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- Conclusions



Coorbital motions: introduction (1)

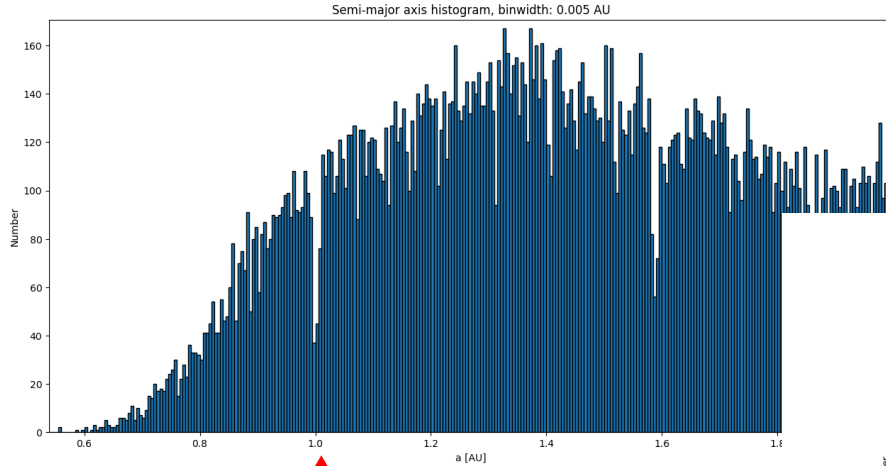


+ compound modes

Coorbital motions: 1:1 mean motion resonance (MMR)

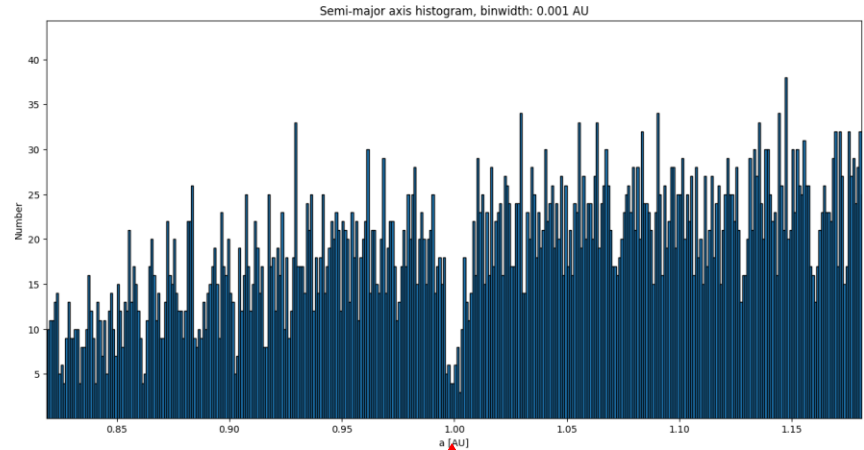
Pan, N., Gallardo, T.: An attempt to build a dynamical catalog of present-day solar system co-orbitals // CMDA, 137:2 (2025)

Coorbital motions: introduction (2)



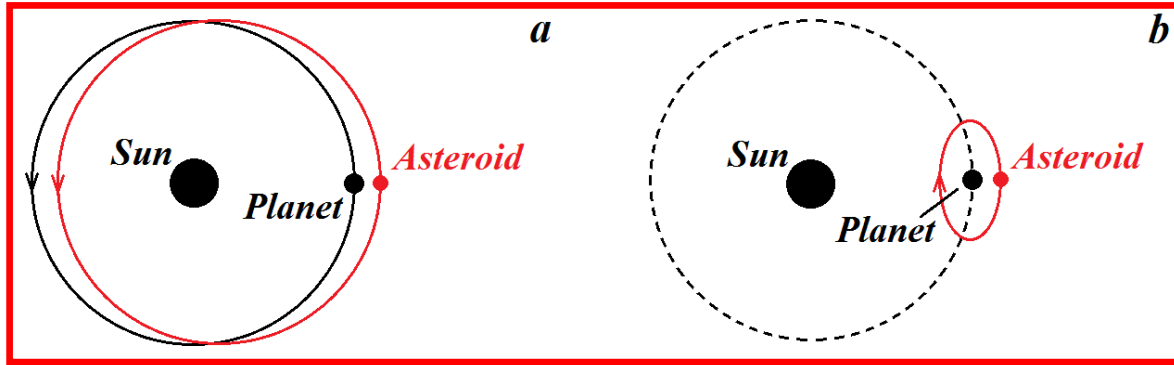
**1:1 MMR
with Earth**

**Histograms characterizing the
distribution of asteroids by the
values of the semi-major axis (2026)**



1:1 MMR with Earth

Coorbital motions: introduction (3)



Quasi-satellites

- a: Sun-centered reference frame with the orientation fixed in the absolute space. The quasi-satellite and the planet move around the Sun with the same orbital period in elliptic and in circular orbits respectively.
- b: Sun-centered frame rotating with the mean orbital motion of the planet.

A.Yu.Kogan (1990): quasi-satellite orbits are the trajectories of restricted three-body problem which are (1) located far beyond the Hill's sphere surrounding the minor primary body and (2) much less distant from it than from the major primary

Sidorenko, V.V., Neishtadt, A.I., Artemyev, A.V., Zelenyi, L.M.: Quasi-satellite orbits in the general context of dynamics in the 1: 1 mean motion resonance: perturbative treatment // CM&DA, 120 (2), 131-162 (2014)

Coorbital motions: introduction (4)

Known Earth Quasi-Satellites

Osculating elements (JD = 2461000.5; November 21, 2025)

Asteroids	a	e	$i, ^\circ$	$\Omega, ^\circ$	$\omega, ^\circ$	$M, ^\circ$
<u>Cardea</u>	1.0011219199	0.1359381515	13.6528649341	38.3866099891	279.3409770102	97.3298092520
2006FV35	1.0014209771	0.3774003454	7.1048695500	179.4594429973	170.7687219999	76.6882034103
2013LX28	1.0016445637	0.4519402136	49.9801822795	76.6636916321	345.7792034121	1.0781488977
<u>Kamo'oalewa</u>	1.0009764525	0.1023678737	7.8030789585	65.6511156082	304.5481116989	46.2346911008
2022YG	1.0036853865	0.1962071528	2.3668935260	273.8744413547	270.9302422129	252.1267110915
2023FW13	0.9993647115	0.1773947238	2.7511935811	69.2526544682	355.0945998274	339.7590806349
2025PN7	1.0009896927	0.1079300842	1.9755405075	112.2503646854	79.8881985606	217.2302471918

Coorbital motions: introduction (5)

Known Earth Quasi-Satellites

[164207 Cardea \(2004 GU9\)](#)

[\(277810\) 2006 FV35](#)

[2013 LX28](#)

[469219 Kamo'oalewa](#)

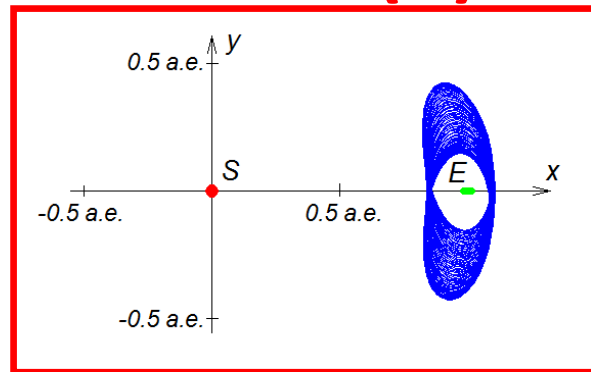
[2022YG](#)

[2023 FW13](#)

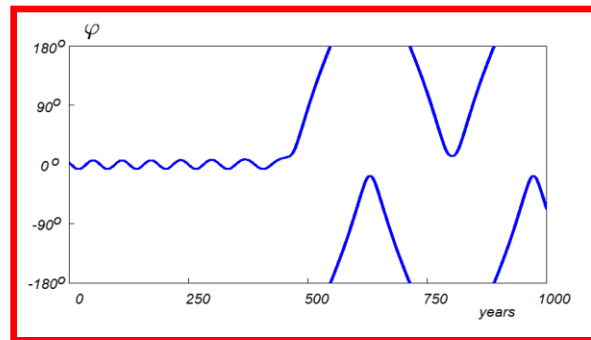
[2025PN7](#)

Target of Tianwen-2
mission (July 2026)

7 objects (August 2026)

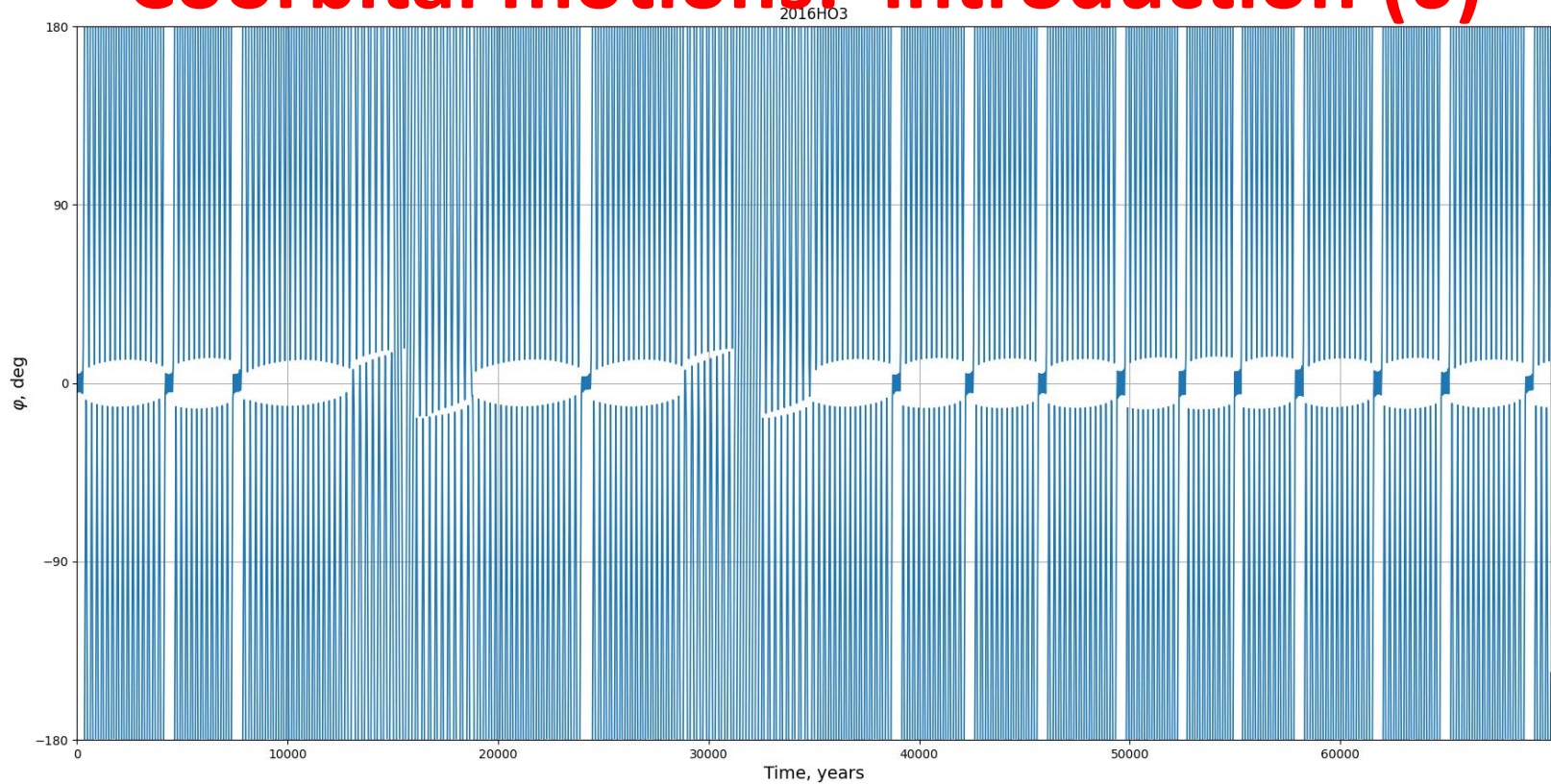


Trajectory of the asteroid Cardea



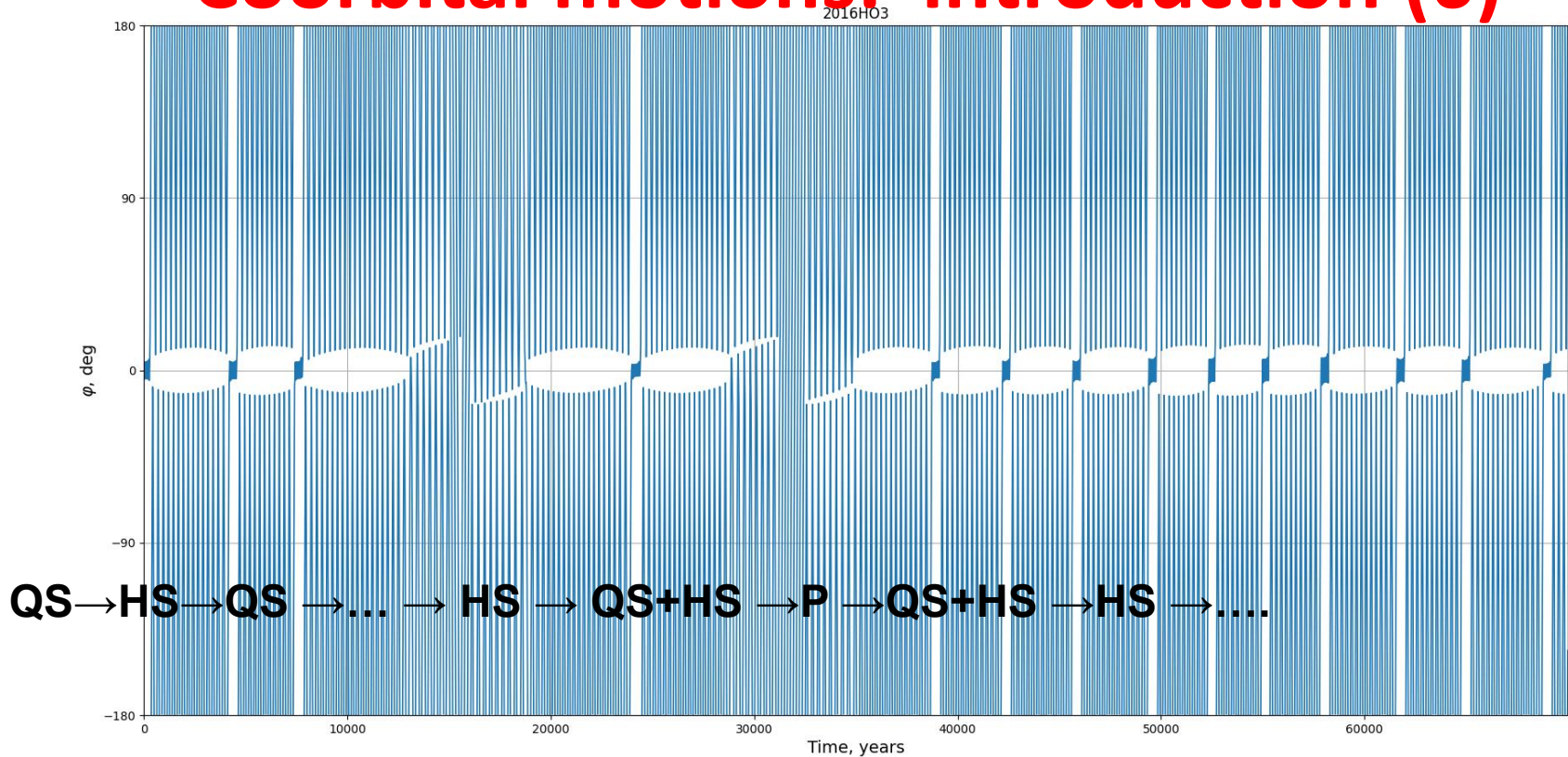
Variation of the resonant phase $\varphi = \lambda - \lambda'$
(λ and λ' are the mean longitudes of
the asteroid and of the planet)

Coorbital motions: introduction (6)



Kamo'oalewa: transitions between different modes

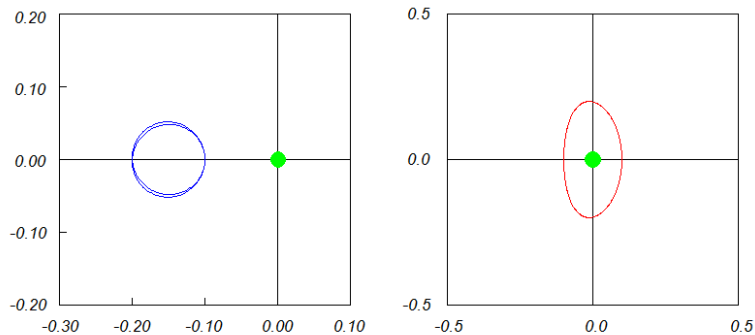
Coorbital motions: introduction (6)



Kamo'oalewa: transitions between different modes

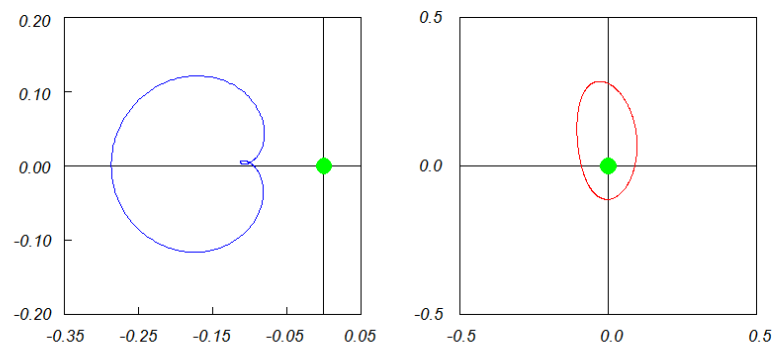
Coorbital motions: introduction (7)

$e = 0.10000$ $\Phi = 0.00000$ (grad)



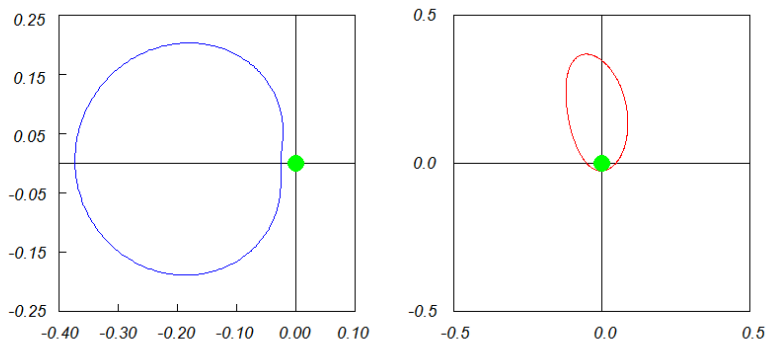
Non-rotating RF vs Rotating RF

$e = 0.10000$ $\Phi = 5.00000$ (grad)



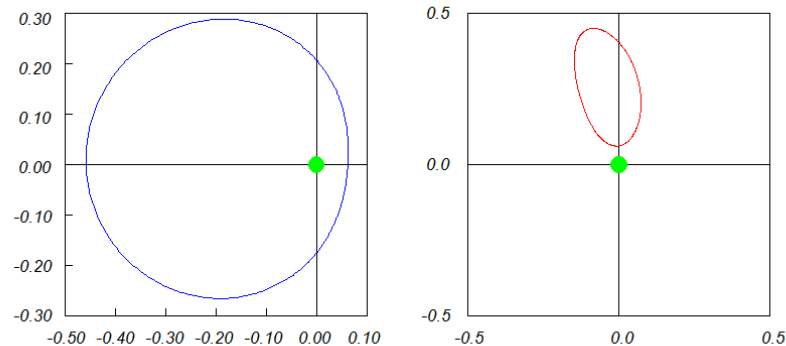
Non-rotating RF vs Rotating RF

$e = 0.10000$ $\Phi = 10.00000$ (grad)



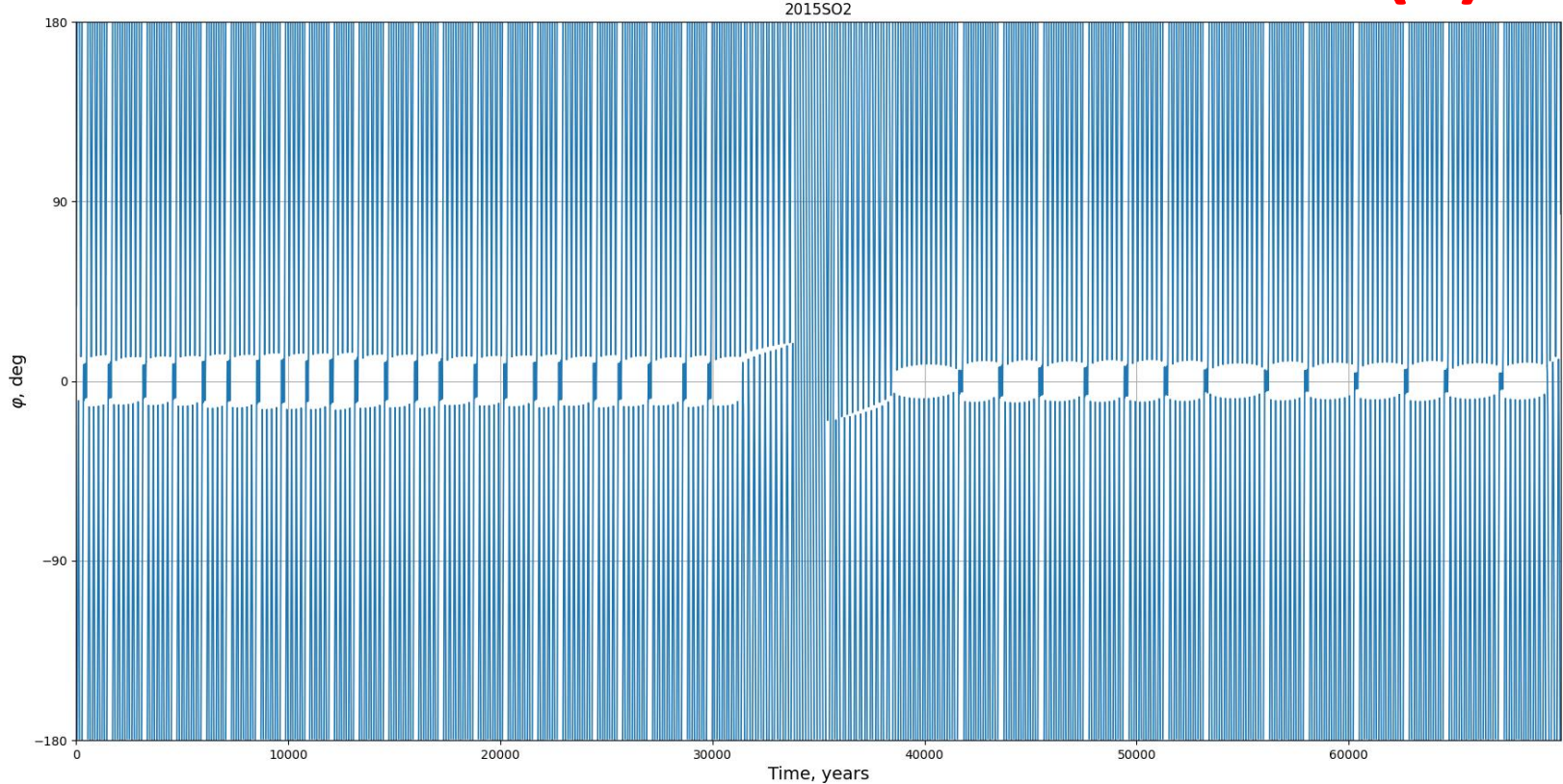
Non-rotating RF vs Rotating RF

$e = 0.10000$ $\Phi = 15.00000$ (grad)



Non-rotating RF vs Rotating RF

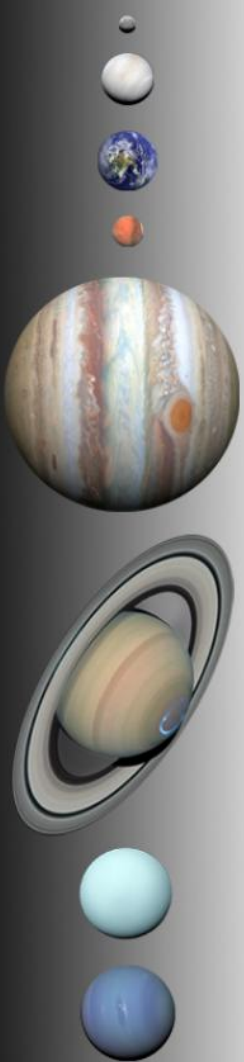
Coorbital motions: introduction (8)



Regular transitions between different modes in the dynamics of asteroid 2015SO2 currently moving in a horseshoe orbit

OUTLINE

- Coorbital motions: introduction
- Adiabatic approximation in studies of resonant motions: main ideas
- Application of AA for studies of coorbital motions
- Conclusions



2. Adiabatic approximation in studies of resonant motions: main ideas (1)

$$e \ll 1, i \ll 1 \longrightarrow$$

Textbooks:
C.D.Murray & S.F.Dermott (1999)
A.Morbidelli(2002)

e, i
are not small



Adiabatic approximation:
Wisdom (1985)

Generalizations:

Henarard & Lemaître (1987), Yokoyama (1996), Sidlichovsky (2005),...



J.Wisdom & V.Sidorenko

2. Adiabatic approximation in studies of resonant motions: main ideas (2)

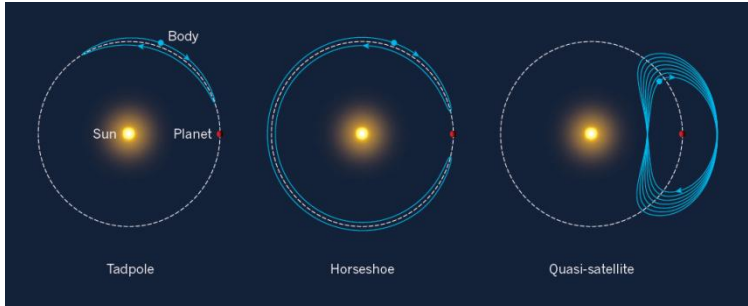
$$\varphi = \lambda_A - \lambda_E$$

Equation, describing the evolution of the resonant phase: $\frac{d^2\varphi}{d\tau^2} + 3\frac{\partial W}{\partial\varphi} = 0,$

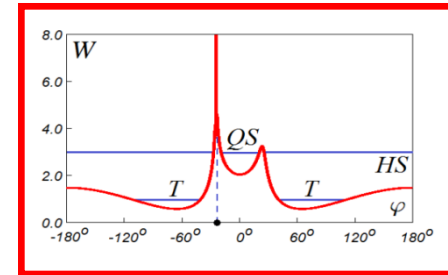
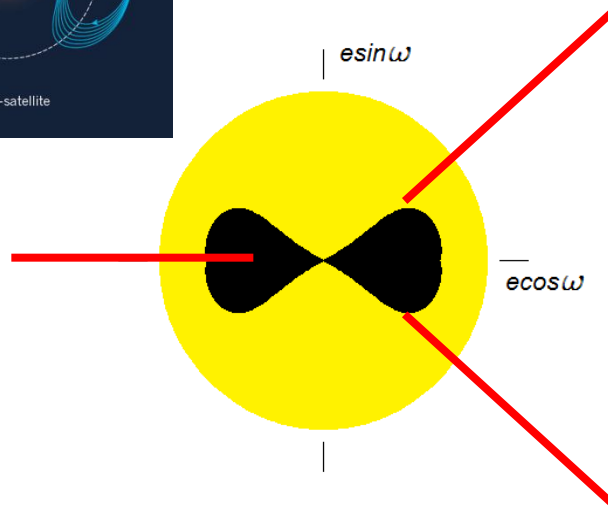
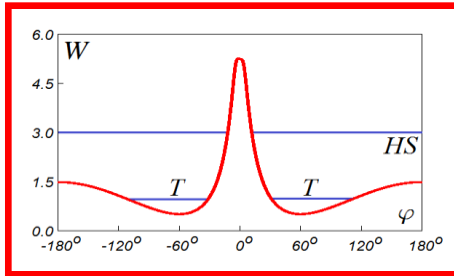
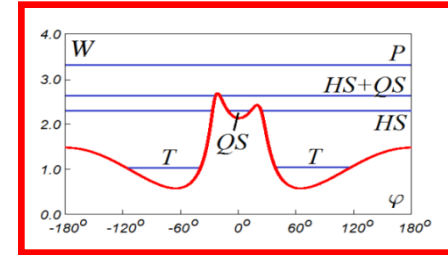
$$W(\varphi, e_A, \omega_A, \sigma) = W_3(\varphi, e_A, \omega_A, \sigma) + \sum_{k=2, k \neq 3}^6 \bar{\mu}_k W_k(e_A, \omega_A, \sigma)$$

$$\sigma = \sqrt{1 - e_A^2} \cdot \cos i_A \quad - \text{ von Zeipel-Lidov-Kozai integral}$$

2. Adiabatic approximation in studies of resonant motions: main ideas (3)



Basic types of orbits
at 1:1 MMR



NCM(1999): W - "ponderomotive potential"
Similar diagram: Christou (2000), Mikkola et al. (2006)

2. Adiabatic approximation in studies of resonant motions (3)

Averaging over the fast subsystem solutions on the level $\Xi = \xi$

$$\frac{de}{d\tau} = -\frac{\varepsilon\sqrt{1-e^2}}{e} \left\langle \frac{\partial W}{\partial \omega} \right\rangle, \quad \frac{d\omega}{d\tau} = \frac{\varepsilon\sqrt{1-e^2}}{e} \left\langle \frac{\partial W}{\partial e} \right\rangle$$

Then it is possible to draw
phase portraits on the
plane e, ω

$$\left\langle \frac{\partial W}{\partial \zeta} \right\rangle = \frac{1}{T(e, \omega, \xi, \sigma)} \int_0^{T(e, \omega, \xi, \sigma)} \frac{\partial W}{\partial \zeta}(e, \omega, \varphi(\tau, e, \omega, \xi, \sigma), \sigma) d\tau$$

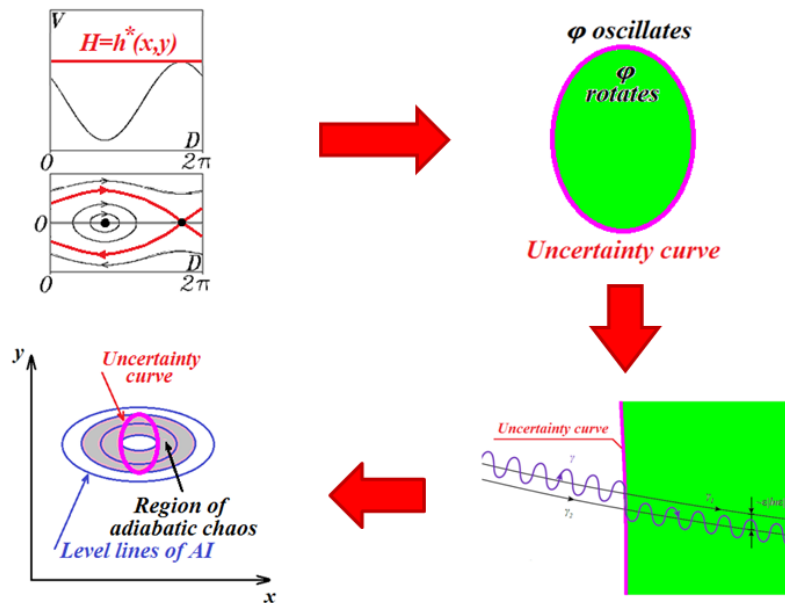
$\zeta = e, \omega$

$$J(e, \omega, \xi, \sigma) = \frac{3}{2\pi} \int_0^{T(e, \omega, \xi, \sigma)} \Phi^2(\tau, e, \omega, \xi, \sigma) d\tau$$

Simplified version of AA: consideration for given ξ, σ level lines $J(e, \omega, \xi, \sigma) = \text{const}$

2. Adiabatic approximation in studies of resonant motions: main ideas (4)

Drawing of phase portraits



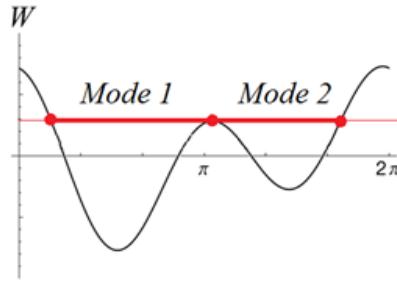
The formation of the region
of adiabatic chaos

The jump between solutions of the averaged
equations in the vicinity of UC

Neishtadt, A.I.: Jumps of the adiabatic invariant on crossing the separatrix and the origin of the 3:1 Kirkwood gap. Sov. Phys. Dokl. **32**, 571–573 (1987)

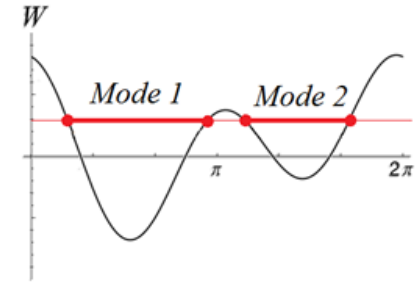
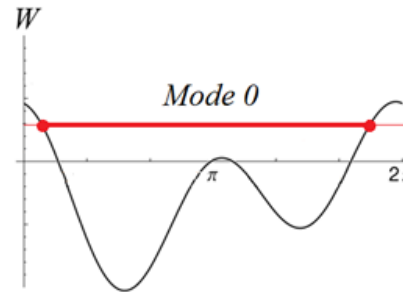
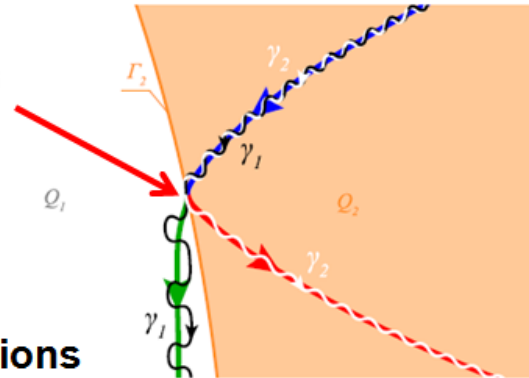
2. Adiabatic approximation: basic ideas

Geometric chaos



Probabilities of transitions
between modes:

Artemyev et al. (2013)



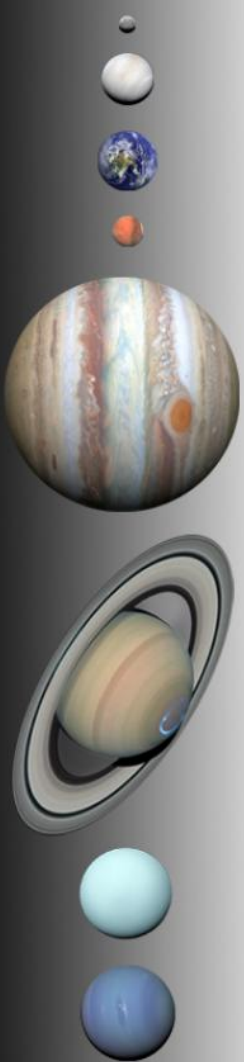
Neishtadt, A., Treschev, D.:
Polymorphisms and adiabatic
chaos. Ergodic Theory &
Dynamical Systems. 31, 259-284
(2011)



Markov chains!

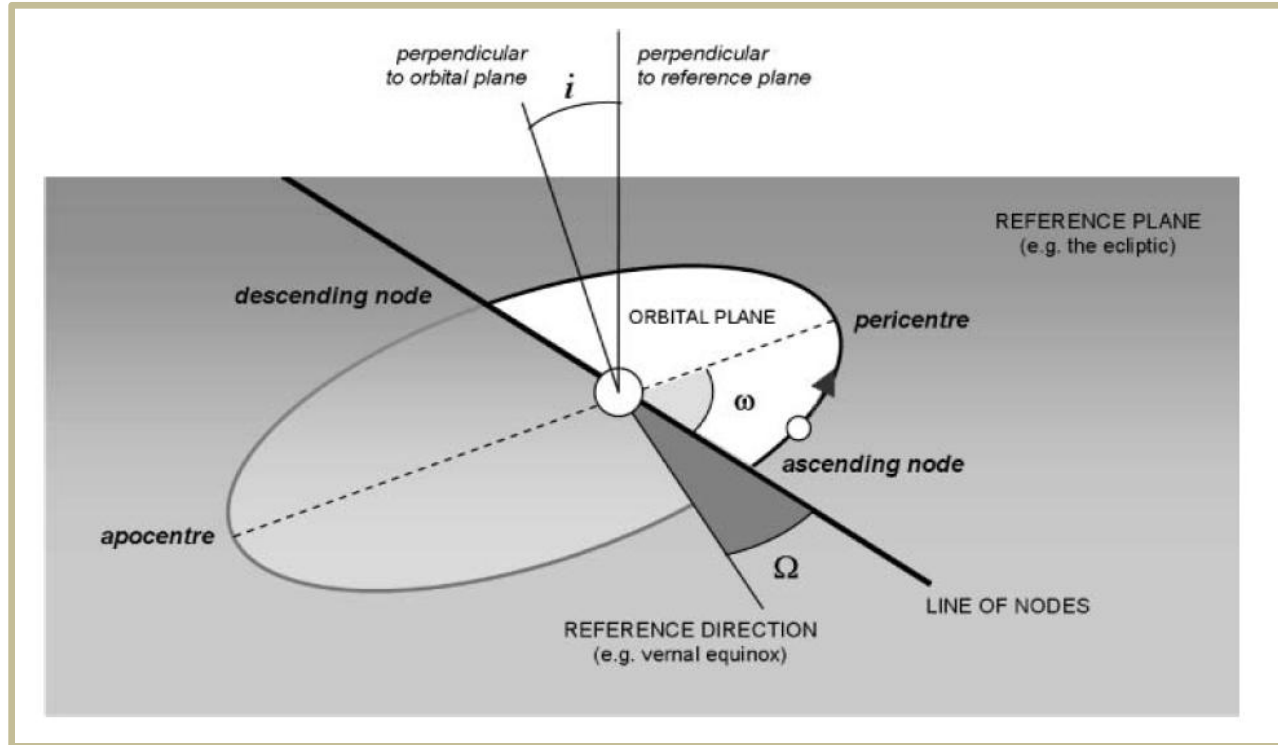
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Application of AA for studies of coorbital motions (1)

Keplerian osculating elements



$$\lambda = \Omega + \omega + l - \text{mean longitude}$$

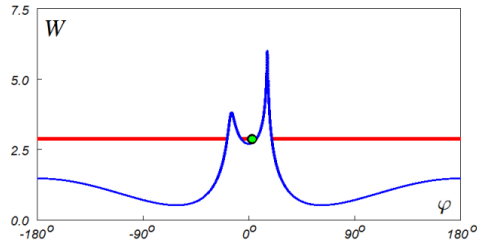
Application of AA for studies of coorbital motions (2)

Estimations of the contribution of different planets to the acceleration of an asteroid in co-orbital motion with the Earth

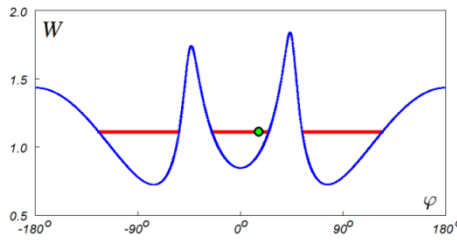
Planet	Typical value of the acceleration (m/sec ²)
Mercury	$\gamma m_{\text{Mercury}} / a_{\text{Earth}}^2 \sim 10^{-9}$
Venus	$\gamma m_{\text{Venus}} / a_{\text{Earth}}^2 \sim 10^{-8}$
Earth(QS)	$\gamma m_{\text{Earth}} / (0.1 a_{\text{Earth}})^2 \sim 10^{-6}$
Earth(HS)	$\gamma m_{\text{Earth}} / a_{\text{Earth}}^2 \sim 10^{-8}$
Mars	$\gamma m_{\text{Mars}} / a_{\text{Mars}}^2 \sim 10^{-9}$
Jupiter	$\gamma m_{\text{Jupiter}} / a_{\text{Jupiter}}^2 \sim 10^{-7}$
Saturn	$\gamma m_{\text{Saturn}} / a_{\text{Saturn}}^2 \sim 10^{-8}$
Uranus	$\gamma m_{\text{Uranus}} / a_{\text{Uranus}}^2 \sim 10^{-9}$
Neptune	$\gamma m_{\text{Neptune}} / a_{\text{Neptune}}^2 \sim 10^{-10}$

Dynamics of the Earth coorbitals: modeling within the framework of N-body problem

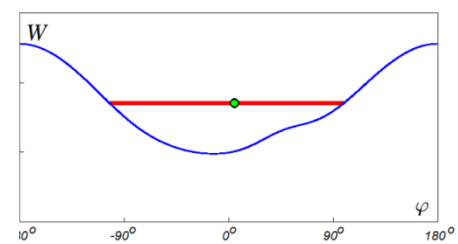
Application of AA for studies of coorbital motions (3)



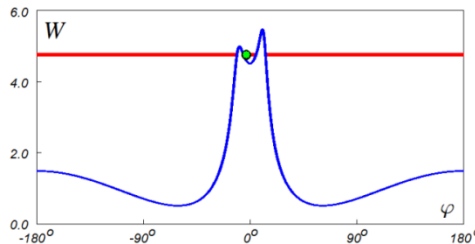
Cardea (2004GU9)
D: 160-360 m



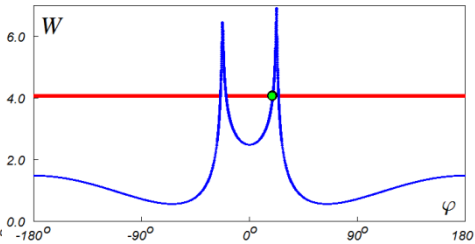
(277810) 2006FV35
D: 140-320 m



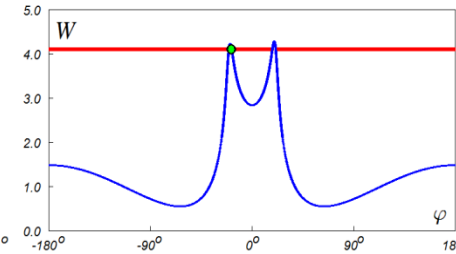
2013LX28
D: 130-300 m



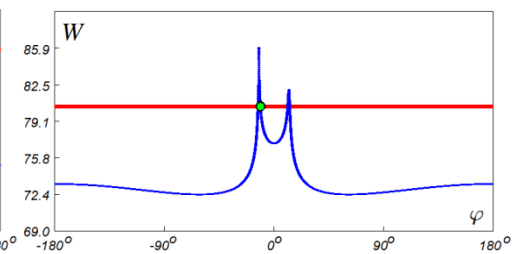
Kamo'oalewa
D: 40-100 m



2022YG
D: 16-30 m



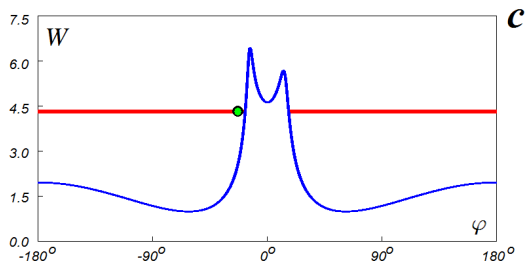
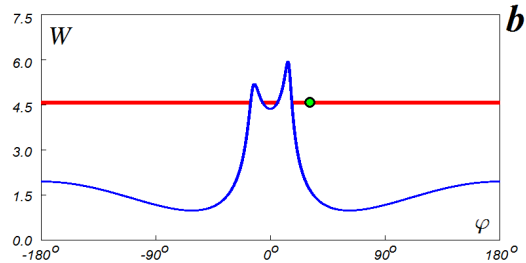
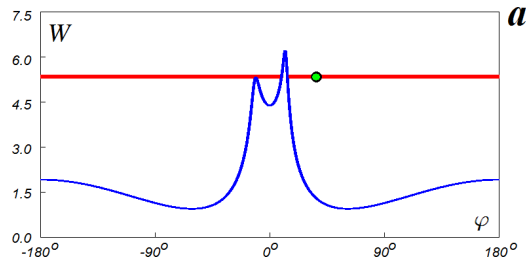
2023FW13
D: 16-30 m



2025PN7
D: 20 m

Known Earth Quasi-Satellites : behavior of the averaged disturbing function

Application of AA for studies of coorbital motions (4)

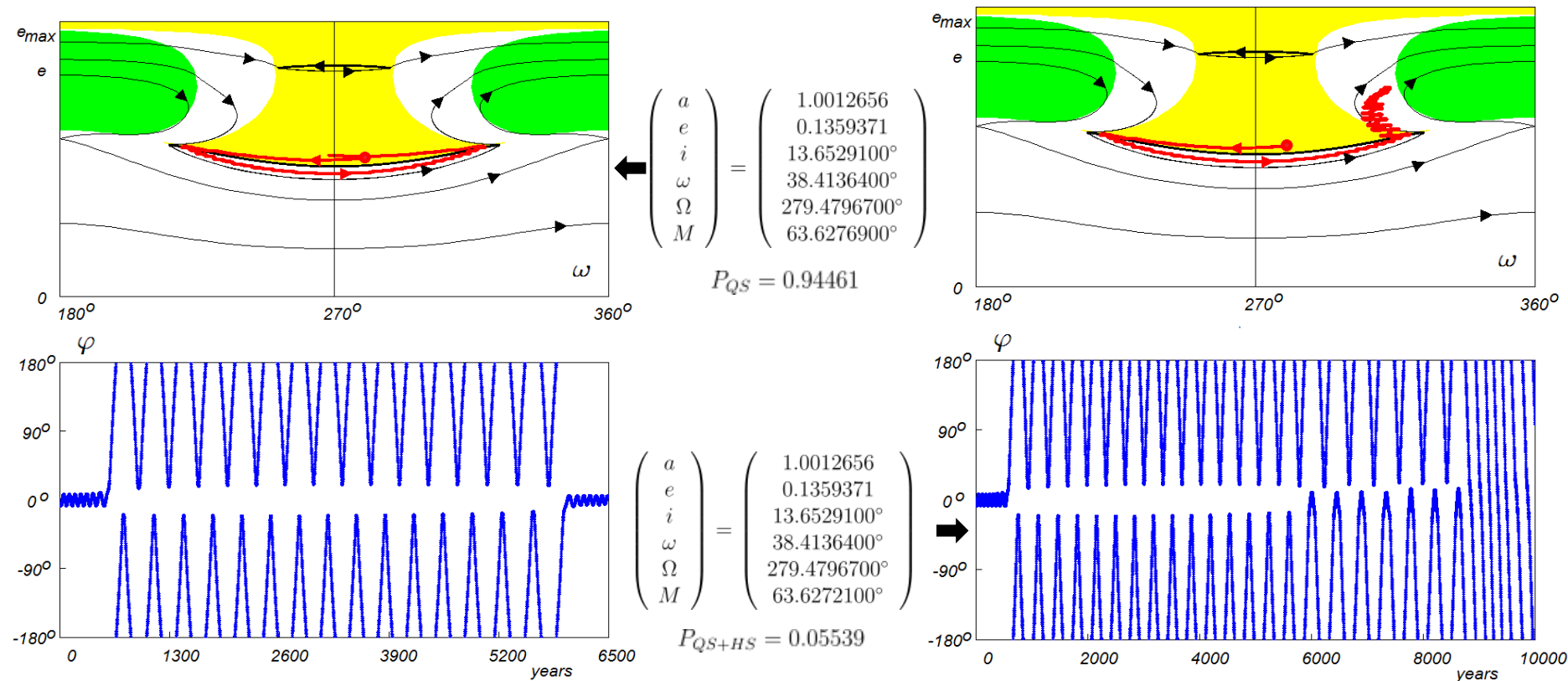


Graphs of averaged disturbing function W for asteroids in HS and QS+HS modes: a – 2015SO2, b - 2016CO246, c – 2020PN

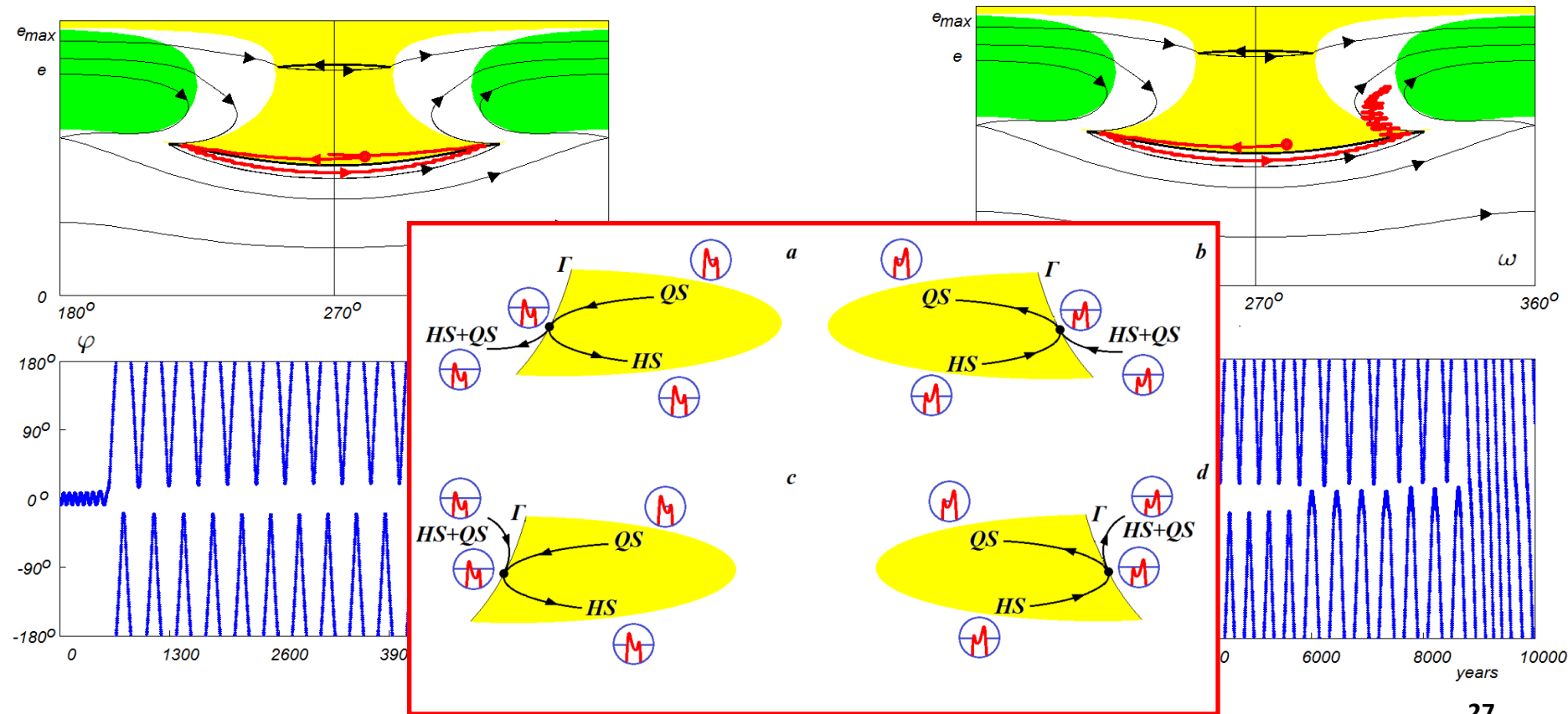
Application of AA for studies of coorbital motions (5)

Dynamically warm coorbitals: a subpopulation at moderate inclinations (5–15 degrees) with $\sigma < 0.277$

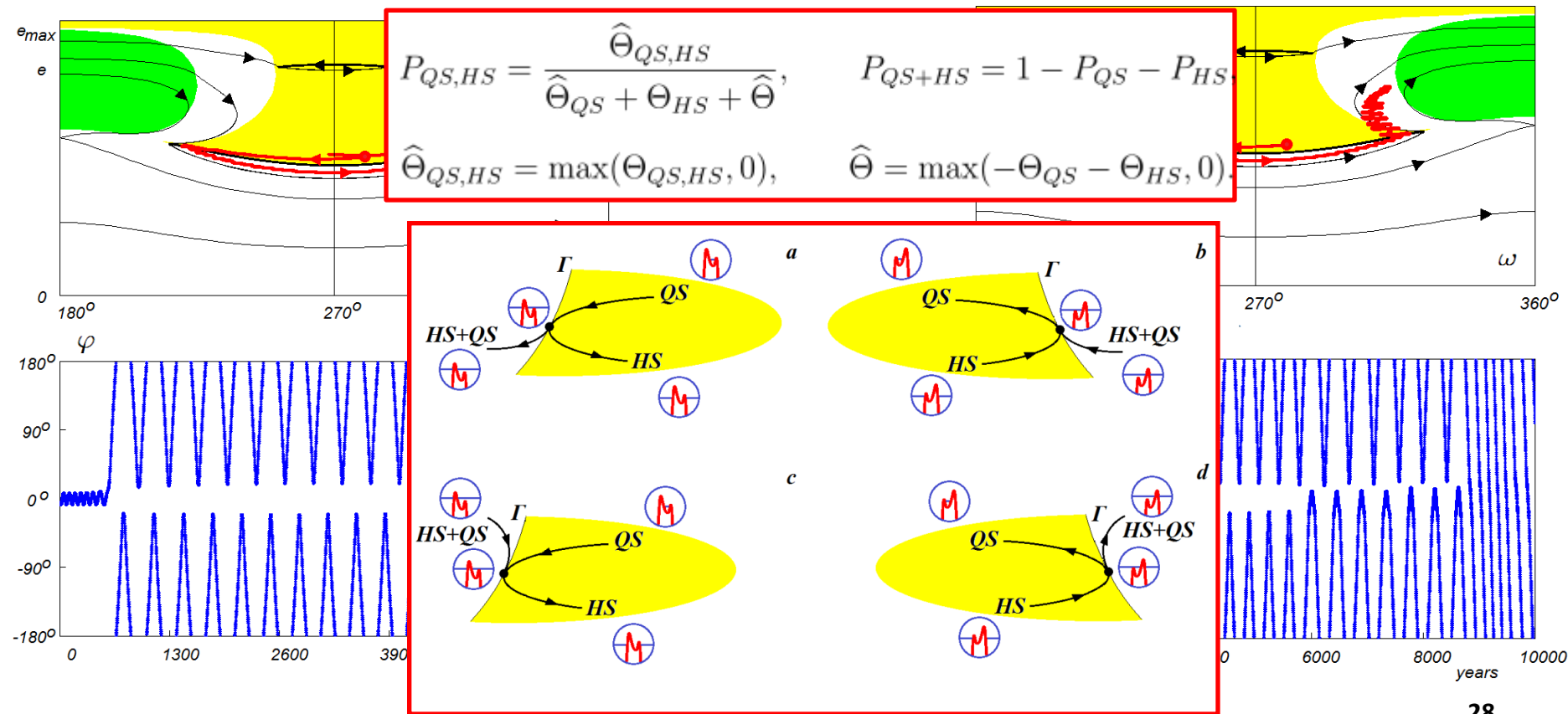
Evolutionary diagram: asteroid (164207) Cardea and its clone



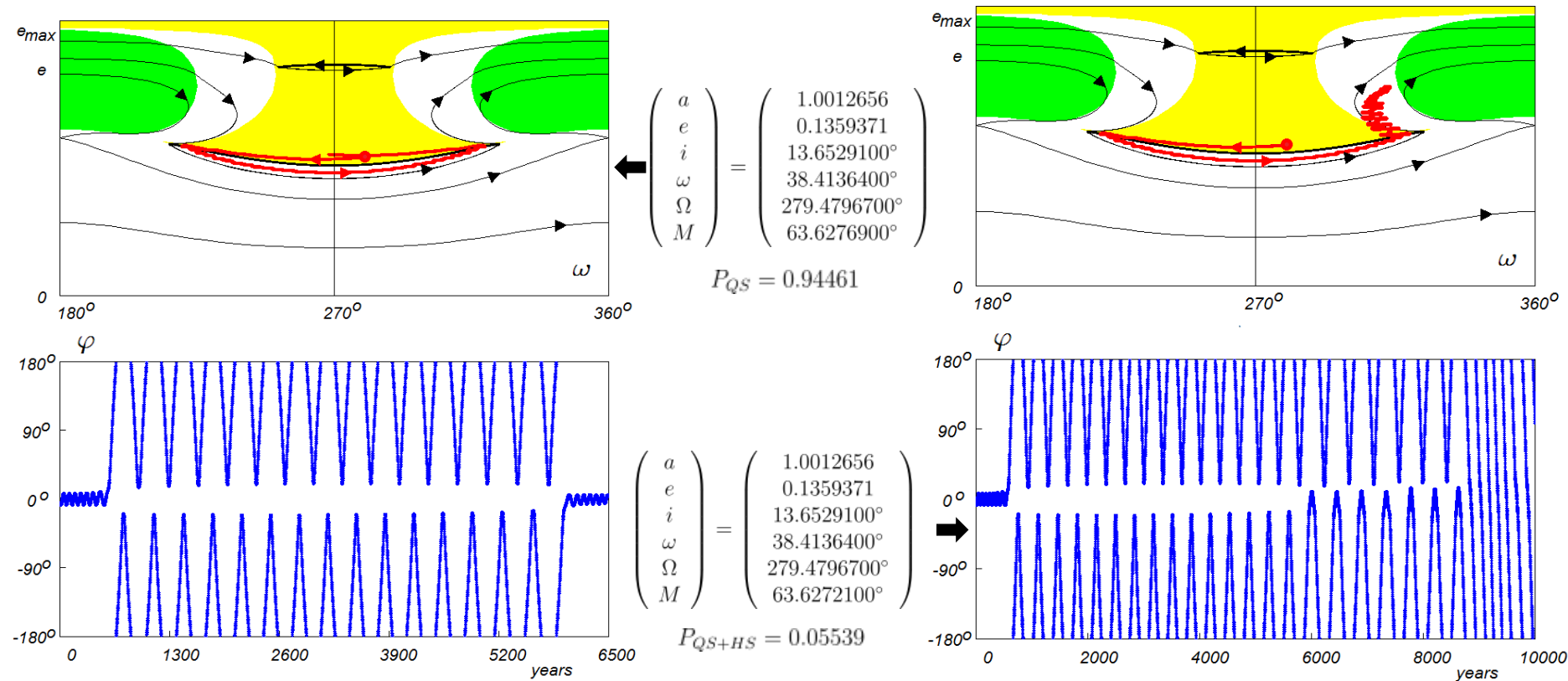
Evolutionary diagram: asteroid (164207) Cardea and its clone



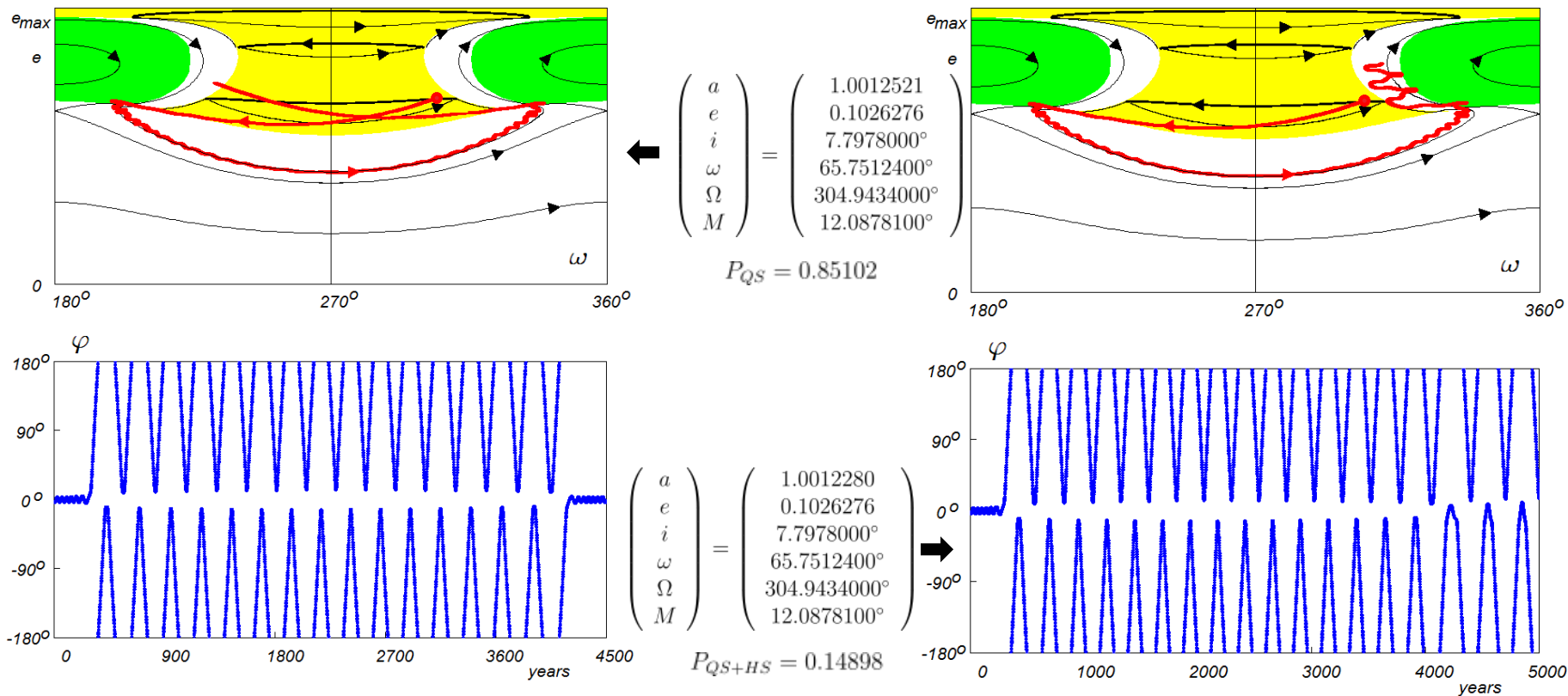
Evolutionary diagram: asteroid (164207) Cardea and its clone



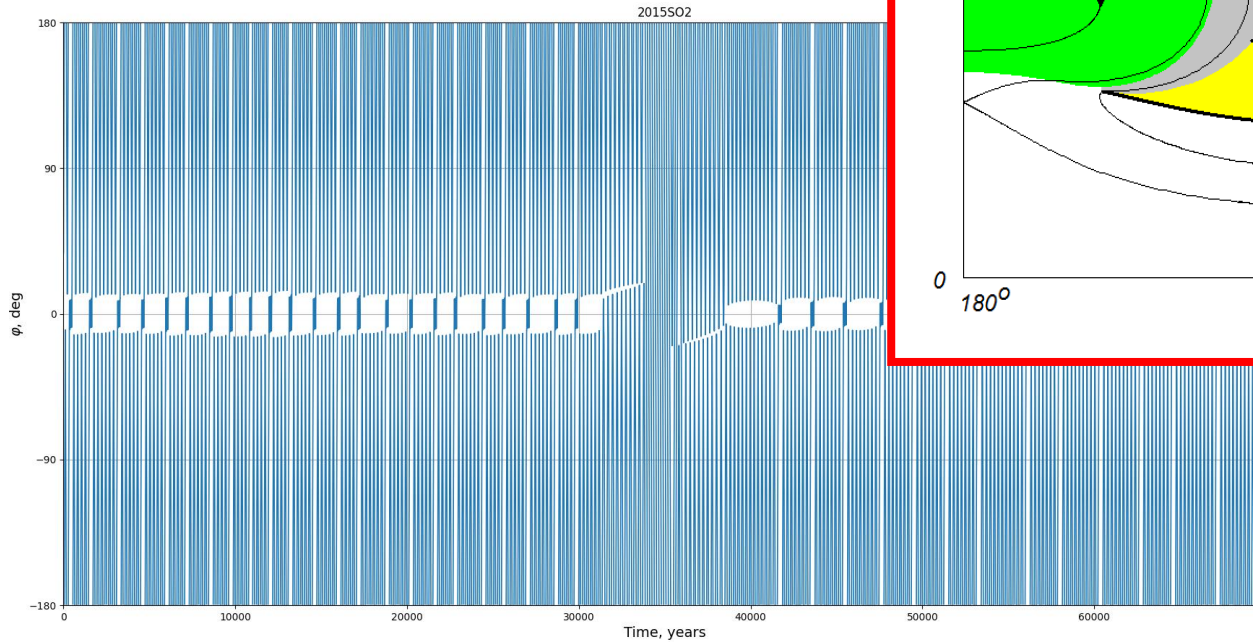
Evolutionary diagram: asteroid (164207) Cardea and its clone



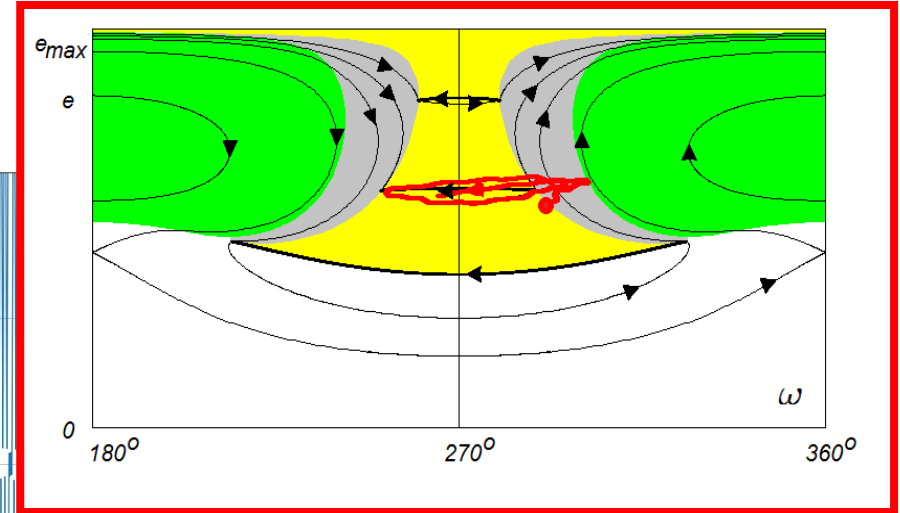
Evolutionary diagram: asteroid (469219) Kamo`oalewa and its clone



Evolutionary diagram: asteroid 2015SO2



Resonance phase behavior

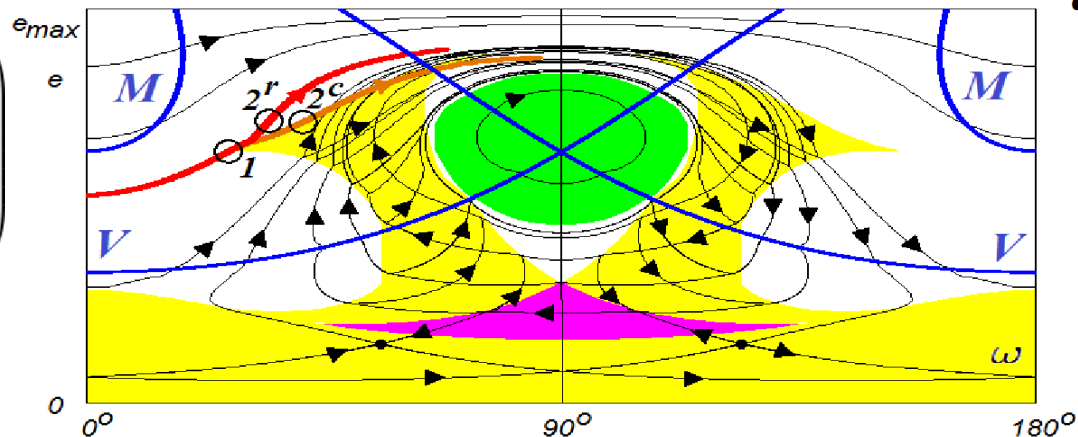


Phase portrait

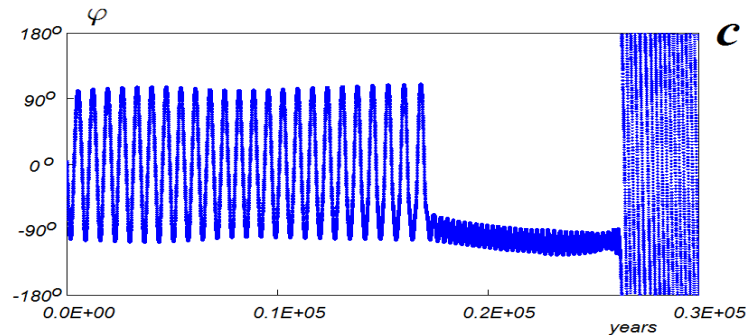
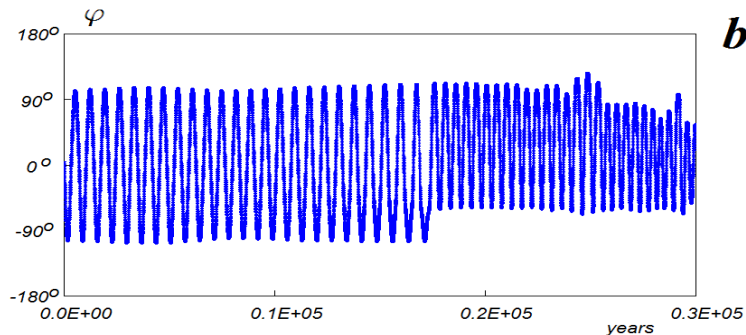
Evolutionary diagram: asteroid 2013LX28 and its clone

$$\begin{pmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ M \end{pmatrix} = \begin{pmatrix} 1.00162550 \\ 0.45197570 \\ 49.980520^\circ \\ 76.663990^\circ \\ 345.775660^\circ \\ 327.807430^\circ \end{pmatrix}$$

$$P_{QS+HS} = 0.34839$$



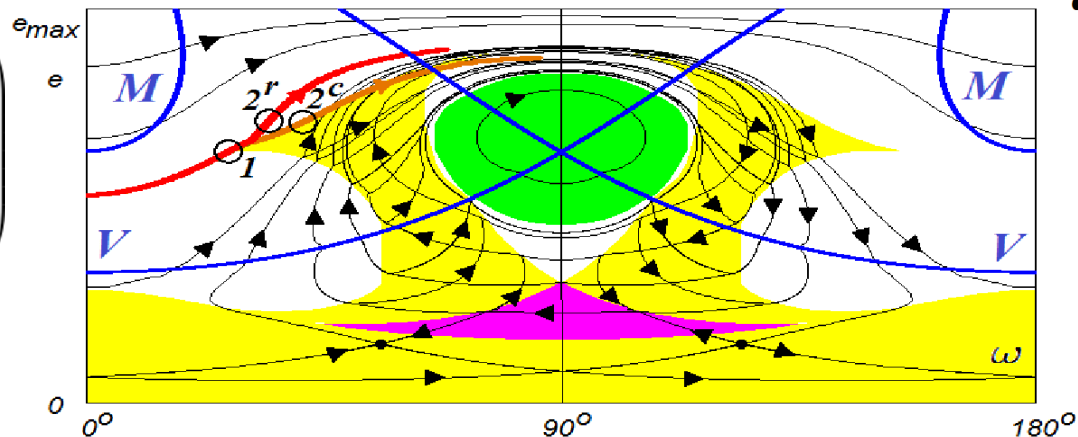
$$\begin{pmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ M \end{pmatrix} = \begin{pmatrix} 1.00161900 \\ 0.45197570 \\ 49.980520^\circ \\ 76.663990^\circ \\ 345.775660^\circ \\ 327.807430^\circ \end{pmatrix}$$



Evolutionary diagram: asteroid 2013LX28 and its clone

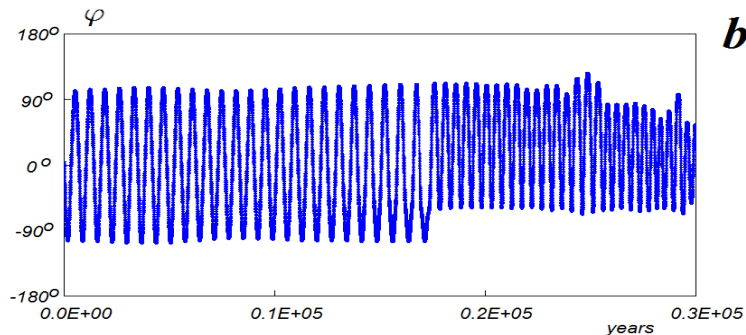
$$\begin{pmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ M \end{pmatrix} = \begin{pmatrix} 1.00162550 \\ 0.45197570 \\ 49.980520^\circ \\ 76.663990^\circ \\ 345.775660^\circ \\ 327.807430^\circ \end{pmatrix}$$

$$P_{QS+HS} = 0.34839$$

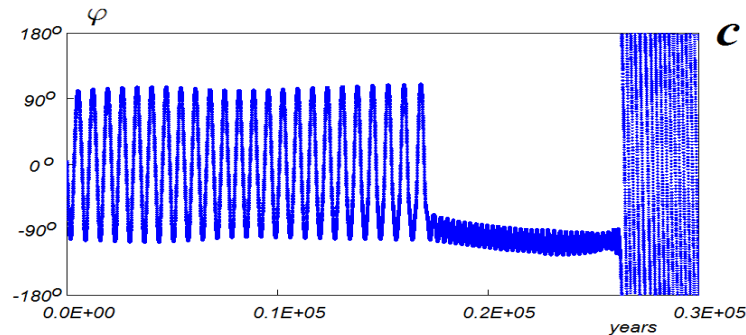


a

$$\begin{pmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ M \end{pmatrix} = \begin{pmatrix} 1.00161900 \\ 0.45197570 \\ 49.980520^\circ \\ 76.663990^\circ \\ 345.775660^\circ \\ 327.807430^\circ \end{pmatrix}$$



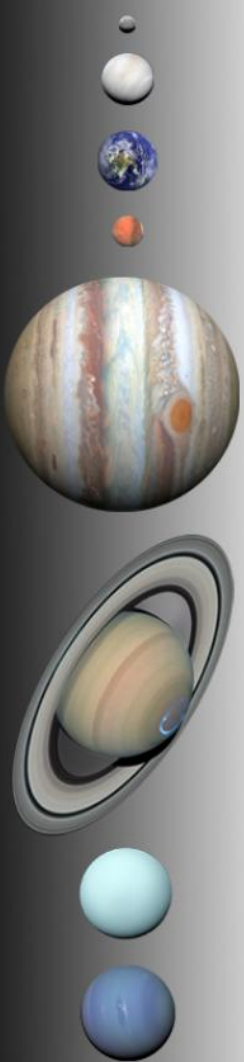
b



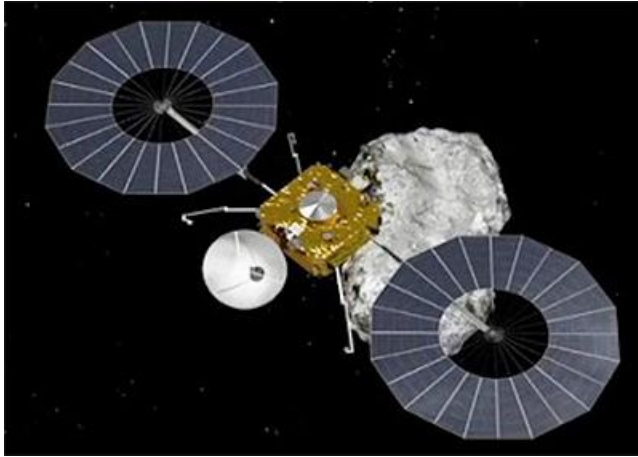
c

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Mission Tianwen-2

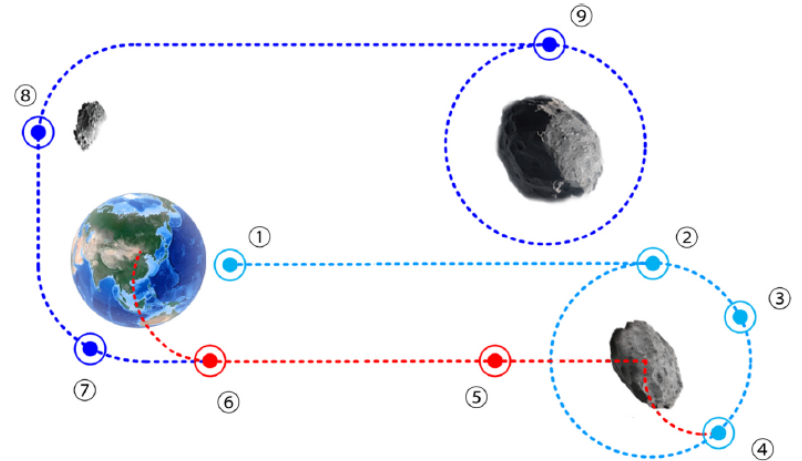


Launch date: 28.05.2025

Launch mass: 2100 kg

Asteroid orbit insertion: 07.06.2025

Return capsule landing: 29.11.2027

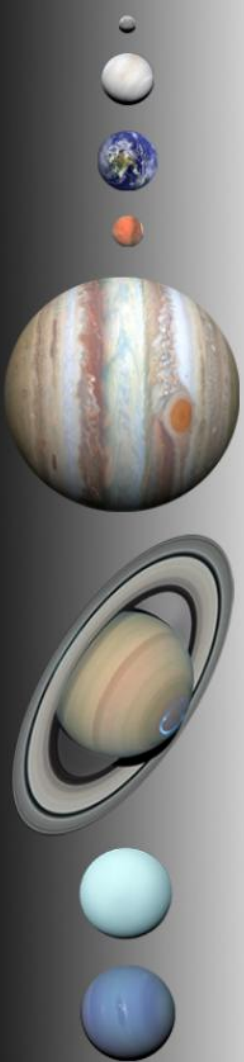


Schematic Diagram of the Tianwen-2 Flight Process. (1) Launch, (2) Rendezvous with 2016 HO₃, (3) Orbital exploration, (4) Attachment sampling, (5) Take-off, (6) Separation of the probe and the return capsule, (7) Gravity-assist maneuver around the Earth, (8) Fly-by a potential asteroid target (to be determined based on the finalized trajectory), (9) Arrive at 311P

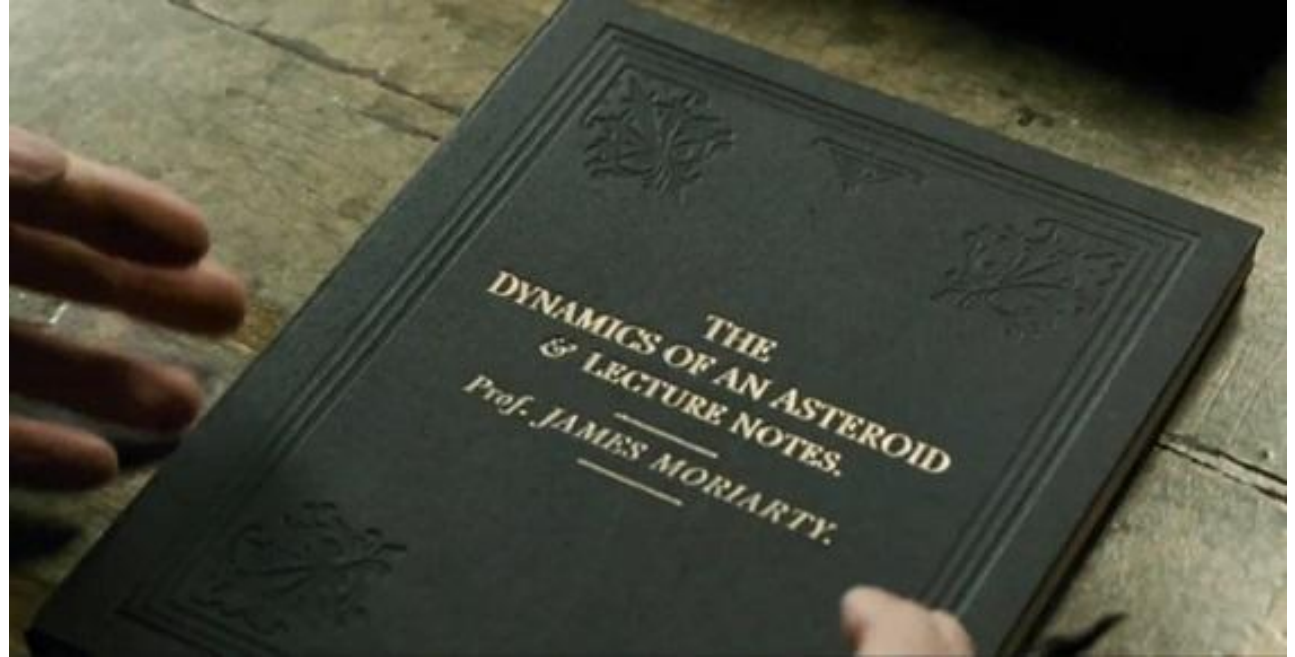
Information and pictures taken from Zhang, R. et al. (Space Science Reviews, 222:11, 2026)

Conclusions

- 1. Coorbitals are important population of small celestial bodies**
- 2. Investigating Earth's coorbitals is crucial both for gaining insight into the dynamical conditions near Earth and for assessing the risk they represent to our planet**
- 3. An important dynamical factor whose significance should be considered in the future is secular resonances**

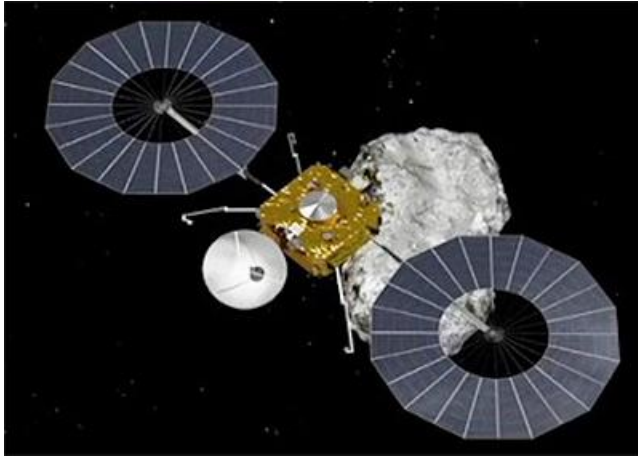


Thank you for your attention!



Sherlock Holmes: A Game of Shadows (2011)

Mission Tianwen-2 (2)



Launch date: 28.05.2025

Launch mass: 2100 kg

Asteroid orbit insertion: 07.06.2025

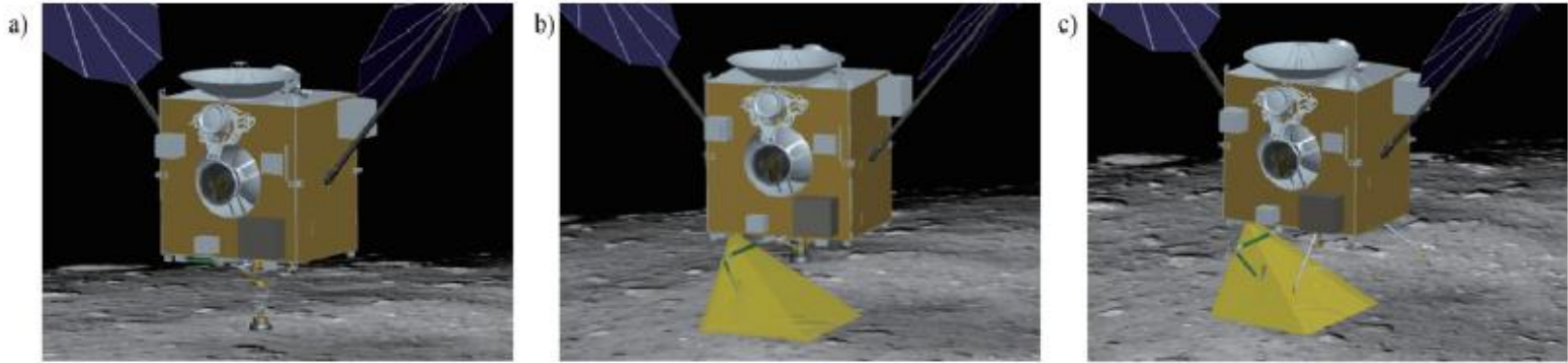
Return capsule landing: 29.11.2027

Scientific payload:

- Visible Infrared Imaging Spectrometer
- Thermal Radiation Spectrometer
- Multispectral Camera
- Medium Field Color Camera
- Detection Radar
- Magnetometer
- Charged and Neutral Particle
- Analyzers
- Ejecta Analyzer
- Narrow Field of View Navigation Sensor
- Laser Integrated Navigation Sensor

Information and pictures taken from Zhang, R. et al. (Space Science Reviews, 222:11, 2026)

Mission Tianwen-2 (3)



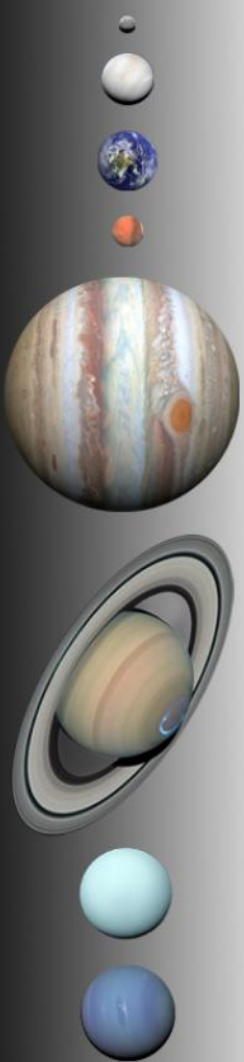
Schematic Diagram of the three sampling methods

a. Hover sampling, b. Touch sampling, c. Attach sampling

Information and pictures taken from Zhang, R. et al. (Space Science Reviews, 222:11, 2026)

OUTLINE

- Quasi-satellite motion: introduction
- Adiabatic approximation in studies of resonant motions: main ideas
- Analysis of QS-motion as 1:1 MMR
- Some remarks about minimoons
- Conclusions



Minimoons

Minimoon: temporarily captured object. The projection onto the ecliptic plane of the radius vector of such an object relative to the Earth's center of mass makes at least one complete rotation (Urrutxua & Bombardelli, 2017)

Typical size : ~ 1 m

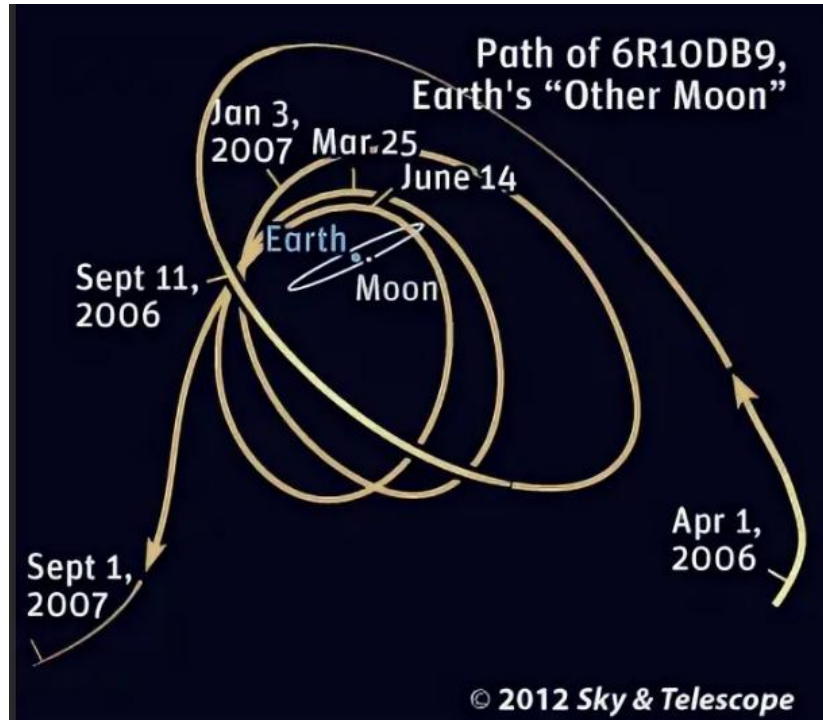
The Earth almost always has one or two minimoons. The mean time of the motion in “minimoon” mode is – 9 months, the mean number of revolutions around the Earth is – 3.

Granvik, M., Vaubaillon, J., Jedicke, R.: The population of natural Earth satellites. Icarus. 218. 262-277 (2012)

Jedicke, R., Bolin, B.T., Bottke, W.F., Chyba, M., Fedorets, G., Granvik, M., Jones, Lynne, Urrutxua, H.: Earth's minimoons: opportunities for science and technology. Frontiers in Astronomy and Space Sciences. 5, paper 13 (2018)

Known Earth Mini-Moons

Asteroid 2006RH120



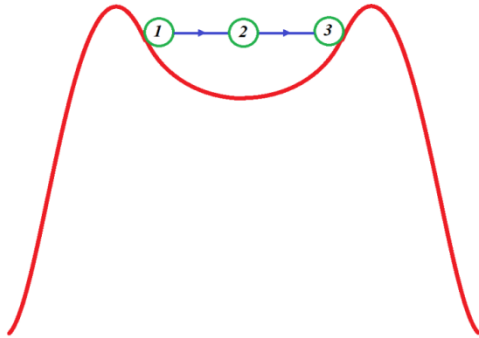
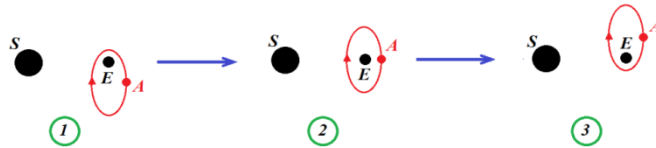
Entered the Hill sphere of Earth in July 2006, left the Hill sphere in July 2007. Four approaches to the Earth.

Size 3-6 m.

Asteroid 2020CD3 (2016-2020, 0.9-1.2 m)

Asteroid 2022NX1

How many quasi-satellites are there around the Earth?



Discoveries of quasi-satellites most often occur at the largest value of the resonant phase!

Typical period of the resonant phase oscillations

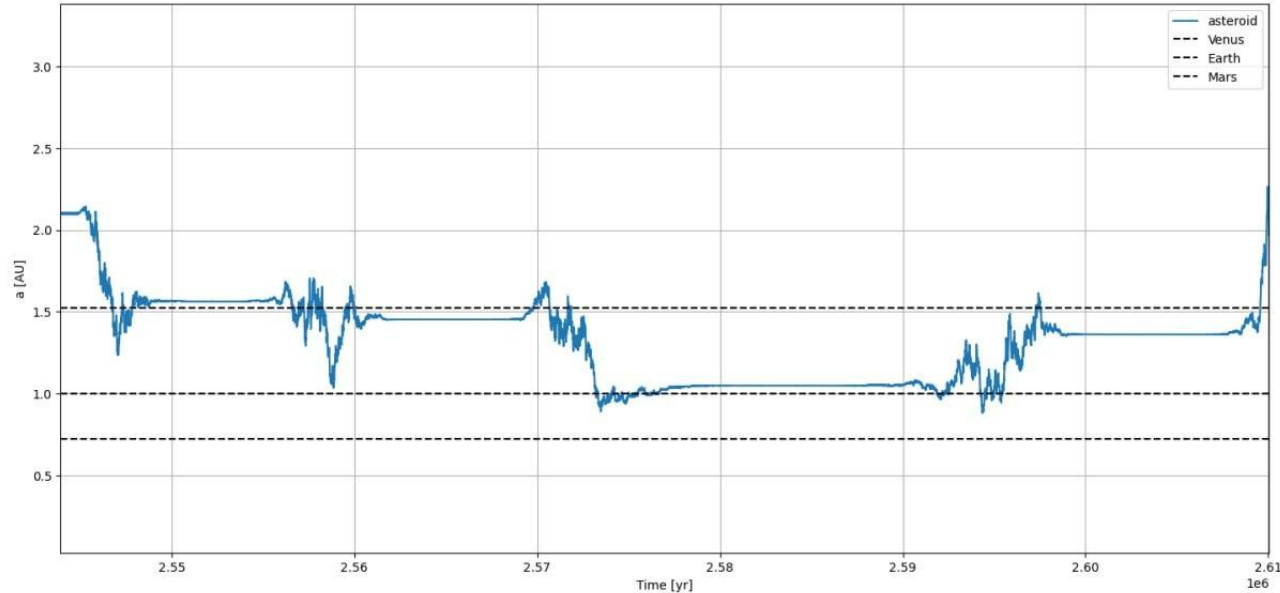
$T \sim 100$ years

Number of quasi-satellites discovered from 2004 to 2024 (20 years): 8

The number of quasi-satellites with D in the range of 10-100 m can be estimated as ~ 20 !

How Earth's co-orbital population arose?

1. Primordial (unlikely);
2. Ejecta from the Moon after a collision
3. Main Belt objects whose orbits were altered by close encounters with Mars, Earth, and Venus.



Phobos-2 space mission (Soviet Union)
Lunched: 12 July 1988
Mars orbital insertion: 29 January 1989
Last contact: 27 March 1989 on quasi-satellite orbit about Phobos
Orbit design: A.Yu. Kogan



MMX (Marsian Moons eXploration)
space mission (Japan)
Scheduled to Launch: 2026
Return soil from Phobos: 2031

